

ANALYSIS OF THE STRONG DECAYS $D_{s3}^*(2860) \rightarrow DK, D^*K$ WITH QCD SUM RULESZhi-Gang Wang¹

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we assign the $D_{s3}^*(2860)$ to be a D-wave $c\bar{s}$ meson, study the hadronic coupling constants $G_{D_{s3}^*(2860)DK}$ and $G_{D_{s3}^*(2860)D^*K}$ with the three-point QCD sum rules, and calculate the partial decay widths $\Gamma(D_{s3}^*(2860) \rightarrow D^*K)$ and $\Gamma(D_{s3}^*(2860) \rightarrow DK)$. The predicted ratio $R = \Gamma(D_{s3}^*(2860) \rightarrow D^*K) / \Gamma(D_{s3}^*(2860) \rightarrow DK) = 0.57 \pm 0.38$ cannot reproduce the experimental value $R = \text{Br}(D_{sJ}^*(2860) \rightarrow D^*K) / \text{Br}(D_{sJ}^*(2860) \rightarrow DK) = 1.10 \pm 0.15 \pm 0.19$.

PACS number: 14.40.Lb, 12.38.Lg

Key words: $D_{s3}^*(2860)$, QCD sum rules

1 Introduction

In 2006, the BaBar collaboration observed the $D_{sJ}^*(2860)$ meson in decays to the final states D^0K^+ and $D^+K_S^0$, the measured mass and width are $(2856.6 \pm 1.5 \pm 5.0) \text{ MeV}$ and $(48 \pm 7 \pm 10) \text{ MeV}$, respectively [1]. In 2009, the BaBar collaboration confirmed the $D_{sJ}^*(2860)$ in the D^*K channel, and measured the ratio R among the branching fractions [2],

$$R = \frac{\text{Br}(D_{sJ}^*(2860) \rightarrow D^*K)}{\text{Br}(D_{sJ}^*(2860) \rightarrow DK)} = 1.10 \pm 0.15 \pm 0.19. \quad (1)$$

The observation of the decays $D_{sJ}^*(2860) \rightarrow D^*K$ rules out the $J^P = 0^+$ assignment, the possible assignments are the 1^3D_3 $c\bar{s}$ meson [3, 4, 5, 6, 7, 8, 9, 10], the $c\bar{s} - cn\bar{s}\bar{n}$ mixing state [11], the dynamically generated $D_1(2420)K$ bound state [12], etc.

In 2014, the LHCb collaboration observed a structure at 2.86 GeV in the $\bar{D}^0 K^-$ mass distribution in the Dalitz plot analysis of the decays $B_s^0 \rightarrow \bar{D}^0 K^- \pi^+$, the structure contains both spin-1 (the $D_{s1}^{*-}(2860)$) and spin-3 (the $D_{s3}^{*-}(2860)$) components [13, 14]. Furthermore, the LHCb collaboration obtained the conclusion that the $D_{sJ}^*(2860)$ observed in the inclusive $e^+e^- \rightarrow \bar{D}^0 K^- X$ production by the BaBar collaboration and in the $pp \rightarrow \bar{D}^0 K^- X$ processes by the LHCb collaboration consists of at least two particles [2, 15].

The QCD sum rules is a powerful theoretical tool in studying the ground state hadrons and has given many successful descriptions of the masses, decay constants, form-factors and hadronic coupling constants, etc [16, 17]. In Ref.[18], we assign the $D_{s3}^*(2860)$ to be a D-wave $c\bar{s}$ meson, and study the mass and decay constant (or the current-meson coupling constant) of the $D_{s3}^*(2860)$ with the QCD sum rules. The predicted mass $M_{D_{s3}^*} = (2.86 \pm 0.10) \text{ GeV}$ is in excellent agreement with the experimental value $M_{D_{s3}^*} = (2860.5 \pm 2.6 \pm 2.5 \pm 6.0) \text{ MeV}$ from the LHCb collaboration [13, 14]. We obtain further support by reproducing the mass of the $D_{s3}^*(2860)$ based on the QCD sum rules.

If we assign the $D_{sJ}^*(2860)$ to be the 1^3D_3 state, the ratio R from the leading order heavy meson effective theory [3], the constituent quark model with quark-meson effective Lagrangians [5], the 3P_0 model [6, 9, 19, 20, 21] and the relativized quark model [22] cannot reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$ [2]. The values of the ratio R from different theoretical methods are shown explicitly in Table 1. From the table, we can see that even in the 3P_0 model the predictions are quite different, as different harmonic oscillator wave-functions are chosen to approximate the mesons' wave-functions.

¹E-mail, zgwang@aliyun.com.

R	Theoretical methods & experimental data
$1.10 \pm 0.15 \pm 0.19$	Experimental value from BaBar [2]
0.39	Leading order heavy meson effective theory [3]
0.40	Constituent quark model with effective Lagrangians [5]
0.59	3P_0 model [6]
0.75	3P_0 model [9]
0.55 – 0.80	3P_0 model [19]
0.68	3P_0 model [20]
0.43	3P_0 model [21]
0.43	Pseudoscalar emission decay model [22]
$1.10 \pm 0.15 \pm 0.19$	Heavy meson effective theory with chiral symmetry breaking corrections [24]

Table 1: The values of the ratio R from different theoretical methods compared to the experimental data.

The $c\bar{q}$ mesons can be sorted in doublets according to the total angular momentum of the light antiquark \vec{s}_ℓ , $\vec{s}_\ell = \vec{s}_{\bar{q}} + \vec{L}$, in the heavy quark limit, where the $\vec{s}_{\bar{q}}$ and \vec{L} are the light antiquark's spin and orbital angular momentum, respectively [23]. For the D-wave mesons, the doublets (D_{s1}^*, D_{s2}) and (D_{s2}', D_{s3}^*) have the spin-parity $J_{s_\ell}^P = (1^-, 2^-)_{\frac{3}{2}}$ and $(2^-, 3^-)_{\frac{5}{2}}$, respectively. The following two-body strong decays can take place,

$$\begin{aligned}
D_{s3}^{*+} &\rightarrow D^{*+}K^0, D^{*0}K^+, D_s^{*+}\eta, D^+K^0, D^0K^+, D_s^+\eta, \\
D_{s2}^+ &\rightarrow D^{*+}K^0, D^{*0}K^+, D_s^{*+}\eta, \\
D_{s2}'^+ &\rightarrow D^{*+}K^0, D^{*0}K^+, D_s^{*+}\eta, \\
D_{s1}^{*+} &\rightarrow D^{*+}K^0, D^{*0}K^+, D_s^{*+}\eta, D^+K^0, D^0K^+, D_s^+\eta.
\end{aligned} \tag{2}$$

In Ref.[24], we assign the $D_{s3}^*(2860)$ and $D_{s1}^*(2860)$ to be the 1^3D_3 and 1^3D_1 $c\bar{s}$ states, respectively, study the strong decays with the heavy meson effective theory by taking into account the chiral symmetry breaking corrections. We can reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$ with suitable hadronic coupling constants \bar{k}_Y^5 and \bar{k}_X^5 , which describe the chiral symmetry breaking corrections. The coupling constant \bar{k}_X^5 in the assignment $D_{sJ}^*(2860) = D_{s1}^*(2860)$ is much larger than the coupling constant \bar{k}_Y^5 in the assignment $D_{sJ}^*(2860) = D_{s3}^*(2860)$. Naively, we expect smaller chiral symmetry breaking corrections, the assignment $D_{sJ}^*(2860) = D_{s3}^*(2860)$ is preferred [24]. On the other hand, if the chiral symmetry breaking effects are small enough to be neglected, we have to include some $D_{s2}^+(2860)$ and $D_{s2}'^+(2860)$ components, as they can only decay to the final states D^*K , which can enhance the ratio R efficiently.

In the article, we take the mass and decay constant (or the current-meson coupling constant) of the $D_{s3}^*(2860)$ from the QCD sum rules as input parameters [18], analyze the vertices $D_{s3}^*(2860)DK$ and $D_{s3}^*(2860)D^*K$ in details to select the pertinent tensor structures, and study the hadronic coupling constants $G_{D_{s3}^*(2860)DK}$ and $G_{D_{s3}^*(2860)D^*K}$ with the three-point QCD sum rules. Then we use the $G_{D_{s3}^*(2860)DK}$ and $G_{D_{s3}^*(2860)D^*K}$ to calculate the partial decay widths $\Gamma(D_{s3}^*(2860) \rightarrow D^*K)$ and $\Gamma(D_{s3}^*(2860) \rightarrow DK)$ and obtain the ratio $R = \Gamma(D_{s3}^*(2860) \rightarrow D^*K) / \Gamma(D_{s3}^*(2860) \rightarrow DK)$, and try to reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$ based on the QCD sum rules so as to obtain additional support for assigning the $D_{sJ}^*(2860)$ to be the $D_{s3}^*(2860)$ [24].

The article is arranged as follows: we derive the QCD sum rules for the hadronic coupling constants $G_{D_{s3}^*(2860)DK}$ and $G_{D_{s3}^*(2860)D^*K}$ in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

2 QCD sum rules for the hadronic coupling constants $G_{D_{s3}^*(2860)DK}$ and $G_{D_{s3}^*(2860)D^*K}$

In the following, we write down the three-point correlation functions $\Pi_{\mu\nu\rho}(p, p')$ and $\Pi_{\sigma\mu\nu\rho}(p, p')$ in the QCD sum rules,

$$\Pi_{\mu\nu\rho}(p, p') = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{i(p-p') \cdot (y-z)} \langle 0 | T \{ J_5(x) J_K(y) J_{\mu\nu\rho}^\dagger(z) \} | 0 \rangle |_{z=0}, \quad (3)$$

$$\Pi_{\sigma\mu\nu\rho}(p, p') = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{i(p-p') \cdot (y-z)} \langle 0 | T \{ J_\sigma(x) J_K(y) J_{\mu\nu\rho}^\dagger(z) \} | 0 \rangle |_{z=0}, \quad (4)$$

$$\begin{aligned} J_5(x) &= \bar{c}(x) i \gamma_5 d(x), \\ J_\sigma(x) &= \bar{c}(x) \gamma_\sigma d(x), \\ J_K(y) &= \bar{d}(y) i \gamma_5 s(y), \\ J_{\mu\nu\rho}(z) &= \bar{c}(z) \left(\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\rho + \gamma_\nu \overleftrightarrow{D}_\rho \overleftrightarrow{D}_\mu + \gamma_\rho \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu \right) s(z), \end{aligned}$$

where the currents $J_5(x)$, $J_\sigma(x)$, $J_K(y)$ and $J_{\mu\nu\rho}(z)$ interpolate the mesons D , D^* , K and $D_{s3}^*(2860)$, respectively, $\overleftrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - ig_s G_\mu - \overleftarrow{\partial}_\mu - ig_s G_\mu$, the G_μ is the gluon field.

The current $J_{\mu\nu\rho}(0)$ has negative parity, and couples potentially to the $J^P = 3^-$ $\bar{c}s$ meson $D_{s3}^*(2860)$. Furthermore, the current $J_{\mu\nu\rho}(0)$ also couples potentially to the $J^P = 2^+$, 1^- , 0^+ $\bar{c}s$ mesons. The current-meson coupling constants or the decay constants $f_{D_{s3}^*}$, $f_{D_{s2}^*}$, $f_{D_{s1}^*}$ and $f_{D_{s0}^*}$ are defined by

$$\langle 0 | J_{\mu\nu\rho}(0) | D_{s3}^*(p) \rangle = f_{D_{s3}^*} \varepsilon_{\mu\nu\rho}(p, s), \quad (5)$$

$$\langle 0 | J_{\mu\nu\rho}(0) | D_{s2}^*(p) \rangle = f_{D_{s2}^*} [p_\mu \varepsilon_{\nu\rho}(p, s) + p_\nu \varepsilon_{\rho\mu}(p, s) + p_\rho \varepsilon_{\mu\nu}(p, s)],$$

$$\langle 0 | J_{\mu\nu\rho}(0) | D_{s1}^*(p) \rangle = f_{D_{s1}^*} [p_\mu p_\nu \varepsilon_\rho(p, s) + p_\nu p_\rho \varepsilon_\mu(p, s) + p_\rho p_\mu \varepsilon_\nu(p, s)],$$

$$\langle 0 | J_{\mu\nu\rho}(0) | D_{s0}^*(p) \rangle = f_{D_{s0}^*} p_\mu p_\nu p_\rho, \quad (6)$$

where the $\varepsilon_{\mu\nu\rho}(p, s)$, $\varepsilon_{\mu\nu}(p, s)$ and $\varepsilon_\mu(p, s)$ are the mesons' polarization vectors with the following properties [25],

$$\begin{aligned} P_{\mu\nu\rho\alpha\beta\sigma} &= \sum_s \varepsilon_{\mu\nu\rho}^*(p, s) \varepsilon_{\alpha\beta\sigma}(p, s) \\ &= \frac{1}{6} (\tilde{g}_{\mu\alpha} \tilde{g}_{\nu\beta} \tilde{g}_{\rho\sigma} + \tilde{g}_{\mu\alpha} \tilde{g}_{\nu\sigma} \tilde{g}_{\rho\beta} + \tilde{g}_{\mu\beta} \tilde{g}_{\nu\alpha} \tilde{g}_{\rho\sigma} + \tilde{g}_{\mu\beta} \tilde{g}_{\nu\sigma} \tilde{g}_{\rho\alpha} + \tilde{g}_{\mu\sigma} \tilde{g}_{\nu\alpha} \tilde{g}_{\rho\beta} + \tilde{g}_{\mu\sigma} \tilde{g}_{\nu\beta} \tilde{g}_{\rho\alpha}) \\ &\quad - \frac{1}{15} (\tilde{g}_{\mu\alpha} \tilde{g}_{\nu\rho} \tilde{g}_{\beta\sigma} + \tilde{g}_{\mu\beta} \tilde{g}_{\nu\rho} \tilde{g}_{\alpha\sigma} + \tilde{g}_{\mu\sigma} \tilde{g}_{\nu\rho} \tilde{g}_{\alpha\beta} + \tilde{g}_{\nu\alpha} \tilde{g}_{\mu\rho} \tilde{g}_{\beta\sigma} + \tilde{g}_{\nu\beta} \tilde{g}_{\mu\rho} \tilde{g}_{\alpha\sigma} + \tilde{g}_{\nu\sigma} \tilde{g}_{\mu\rho} \tilde{g}_{\alpha\beta} \\ &\quad + \tilde{g}_{\rho\alpha} \tilde{g}_{\mu\nu} \tilde{g}_{\beta\sigma} + \tilde{g}_{\rho\beta} \tilde{g}_{\mu\nu} \tilde{g}_{\alpha\sigma} + \tilde{g}_{\rho\sigma} \tilde{g}_{\mu\nu} \tilde{g}_{\alpha\beta}), \end{aligned} \quad (7)$$

$$P_{\mu\nu\alpha\beta} = \sum_s \varepsilon_{\mu\nu}^*(p, s) \varepsilon_{\alpha\beta}(p, s) = \frac{\tilde{g}_{\mu\alpha} \tilde{g}_{\nu\beta} + \tilde{g}_{\mu\beta} \tilde{g}_{\nu\alpha}}{2} - \frac{\tilde{g}_{\mu\nu} \tilde{g}_{\alpha\beta}}{3}, \quad (8)$$

$$\tilde{g}_{\mu\nu} = \sum_s \varepsilon_\mu^*(p, s) \varepsilon_\nu(p, s) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}. \quad (9)$$

At the phenomenological side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_5(x)$, $J_\sigma(x)$, $J_K(y)$ and $J_{\mu\nu\rho}(z)$ into the correlation functions $\Pi_{\mu\nu\rho}(p, p')$ and $\Pi_{\sigma\mu\nu\rho}(p, p')$ to obtain the hadronic representation [16, 17].

We isolate all the ground state contributions and write them down explicitly,

$$\begin{aligned}
\Pi_{\mu\nu\rho}(p, p') = & \frac{f_D M_D^2 f_K M_K^2 f_{D_{s3}^*} G_{D_{s3}^* DK}(q^2)}{(m_c + m_d)(m_d + m_s) (M_D^2 - p'^2) (M_K^2 - q^2) (M_{D_{s3}^*}^2 - p^2)} \\
& \left\{ \frac{[\lambda (M_{D_{s3}^*}^2, M_D^2, q^2) + 10 M_{D_{s3}^*}^2 M_D^2] (M_{D_{s3}^*}^2 + M_D^2 - q^2)}{20 M_{D_{s3}^*}^6} p_\mu p_\nu p_\rho \right. \\
& + \frac{\lambda (M_{D_{s3}^*}^2, M_D^2, q^2) (M_{D_{s3}^*}^2 + M_D^2 - q^2)}{40 M_{D_{s3}^*}^4} (p_\mu g_{\nu\rho} + p_\nu g_{\mu\rho} + p_\rho g_{\mu\nu}) \\
& - \frac{\lambda (M_{D_{s3}^*}^2, M_D^2, q^2)}{20 M_{D_{s3}^*}^2} (p'_\mu g_{\nu\rho} + p'_\nu g_{\mu\rho} + p'_\rho g_{\mu\nu}) \\
& - \frac{\lambda (M_{D_{s3}^*}^2, M_D^2, q^2) + 5 M_{D_{s3}^*}^2 M_D^2}{5 M_{D_{s3}^*}^4} (p'_\mu p_\nu p_\rho + p'_\nu p_\mu p_\rho + p'_\rho p_\mu p_\nu) \\
& \left. + \frac{M_{D_{s3}^*}^2 + M_D^2 - q^2}{2 M_{D_{s3}^*}^2} (p'_\mu p'_\nu p_\rho + p'_\nu p'_\rho p_\mu + p'_\rho p'_\mu p_\nu) - p'_\mu p'_\nu p'_\rho \right\} \\
& + \frac{f_D M_D^2 f_K M_K^2 f_{D_{s2}^*} G_{D_{s2}^* DK}(q^2)}{(m_c + m_d)(m_d + m_s) (M_D^2 - p'^2) (M_K^2 - q^2) (M_{D_{s2}^*}^2 - p^2)} \\
& \left\{ \frac{\lambda (M_{D_{s2}^*}^2, M_D^2, q^2) + 6 M_{D_{s2}^*}^2 M_D^2}{2 M_{D_{s2}^*}^4} p_\mu p_\nu p_\rho \right. \\
& + \frac{\lambda (M_{D_{s2}^*}^2, M_D^2, q^2)}{12 M_{D_{s2}^*}^2} (p_\mu g_{\nu\rho} + p_\nu g_{\mu\rho} + p_\rho g_{\mu\nu}) \\
& + (p'_\mu p'_\nu p_\rho + p'_\nu p'_\rho p_\mu + p'_\rho p'_\mu p_\nu) \\
& \left. - \frac{M_{D_{s2}^*}^2 + M_D^2 - q^2}{M_{D_{s2}^*}^2} (p'_\mu p_\nu p_\rho + p'_\nu p_\mu p_\rho + p'_\rho p_\mu p_\nu) \right\} \\
& + \frac{f_D M_D^2 f_K M_K^2 f_{D_{s1}^*} G_{D_{s1}^* DK}(q^2)}{(m_c + m_d)(m_d + m_s) (M_D^2 - p'^2) (M_K^2 - q^2) (M_{D_{s1}^*}^2 - p^2)} \\
& \left\{ \frac{3 (M_{D_{s1}^*}^2 + M_D^2 - q^2)}{2 M_{D_{s1}^*}^2} p_\mu p_\nu p_\rho - (p'_\mu p_\nu p_\rho + p'_\nu p_\mu p_\rho + p'_\rho p_\mu p_\nu) \right\} \\
& + \frac{f_D M_D^2 f_K M_K^2 f_{D_{s0}^*} G_{D_{s0}^* DK}(q^2)}{(m_c + m_d)(m_d + m_s) (M_D^2 - p'^2) (M_K^2 - q^2) (M_{D_{s0}^*}^2 - p^2)} p_\mu p_\nu p_\rho + \dots \quad (10)
\end{aligned}$$

$$\begin{aligned}
\Pi_{\sigma\mu\nu\rho}(p, p') = & \frac{f_{D^*} M_{D^*} f_K M_K^2 f_{D_{s3}^*} G_{D_{s3}^* D^* K}(q^2)}{(m_d + m_s) (M_{D^*}^2 - p'^2) (M_K^2 - q^2) (M_{D_{s3}^*}^2 - p^2)} \\
& \left\{ \frac{\lambda(M_{D_{s3}^*}^2, M_{D^*}^2, q^2)}{60 M_{D_{s3}^*}^2} (g_{\mu\nu} \varepsilon_{\sigma\rho\lambda\tau} p^\lambda p'^\tau + g_{\mu\rho} \varepsilon_{\sigma\nu\lambda\tau} p^\lambda p'^\tau + g_{\nu\rho} \varepsilon_{\sigma\mu\lambda\tau} p^\lambda p'^\tau) \right. \\
& + \frac{\lambda(M_{D_{s3}^*}^2, M_{D^*}^2, q^2) + 5 M_{D_{s3}^*}^2 M_{D^*}^2}{15 M_{D_{s3}^*}^4} \\
& (\varepsilon_{\sigma\rho\lambda\tau} p_\mu p_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p_\mu p_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p_\nu p_\rho p^\lambda p'^\tau) \\
& - \frac{M_{D_{s3}^*}^2 + M_{D^*}^2 - q^2}{6 M_{D_{s3}^*}^2} (\varepsilon_{\sigma\rho\lambda\tau} p'_\mu p_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\rho\lambda\tau} p_\mu p'_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p'_\mu p_\rho p^\lambda p'^\tau \\
& + \varepsilon_{\sigma\nu\lambda\tau} p_\mu p'_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p'_\nu p_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p_\nu p'_\rho p^\lambda p'^\tau) \\
& \left. + \frac{1}{3} (\varepsilon_{\sigma\rho\lambda\tau} p'_\mu p'_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p'_\mu p'_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p'_\nu p'_\rho p^\lambda p'^\tau) \right\} \\
& + \frac{f_{D^*} M_{D^*} f_K M_K^2 f_{D_{s2}^*} G_{D_{s2}^* D^* K}(q^2)}{(m_d + m_s) (M_{D^*}^2 - p'^2) (M_K^2 - q^2) (M_{D_{s2}^*}^2 - p^2)} \\
& \left\{ -\frac{1}{2} (\varepsilon_{\sigma\rho\lambda\tau} p'_\mu p_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\rho\lambda\tau} p_\mu p'_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p'_\mu p_\rho p^\lambda p'^\tau \right. \\
& + \varepsilon_{\sigma\nu\lambda\tau} p_\mu p'_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p'_\nu p_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p_\nu p'_\rho p^\lambda p'^\tau) \\
& \left. + \frac{M_{D_{s2}^*}^2 + M_{D^*}^2 - q^2}{2 M_{D_{s2}^*}^2} (\varepsilon_{\sigma\rho\lambda\tau} p_\mu p_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p_\mu p_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p_\nu p_\rho p^\lambda p'^\tau) \right\} \\
& + \frac{f_{D^*} M_{D^*} f_K M_K^2 f_{D_{s1}^*} G_{D_{s1}^* D^* K}(q^2)}{(m_d + m_s) (M_{D^*}^2 - p'^2) (M_K^2 - q^2) (M_{D_{s1}^*}^2 - p^2)} \\
& (\varepsilon_{\sigma\rho\lambda\tau} p_\mu p_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p_\mu p_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p_\nu p_\rho p^\lambda p'^\tau) + \dots, \tag{11}
\end{aligned}$$

where the \dots denotes the contributions come from the higher resonances and continuum states, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$, the decay constants f_D , f_{D^*} , f_K and the hadronic coupling constants $G_{D_{s3}^* DK}$, $G_{D_{s2}^* DK}$, $G_{D_{s1}^* DK}$, $G_{D_{s0}^* DK}$, $G_{D_{s3}^* D^* K}$, $G_{D_{s2}^* D^* K}$, $G_{D_{s1}^* D^* K}$ are defined by

$$\begin{aligned}
\langle 0 | J_5(0) | D(p') \rangle &= \frac{f_D M_D^2}{m_c + m_d}, \\
\langle 0 | J_\sigma(0) | D^*(p') \rangle &= f_{D^*} M_{D^*} \varepsilon_\sigma(p', s), \\
\langle 0 | J_K(0) | K(q) \rangle &= \frac{f_K M_K^2}{m_s + m_d}, \tag{12}
\end{aligned}$$

$$\begin{aligned}
\langle D(p') K(q) | D_{s3}^*(p) \rangle &= G_{D_{s3}^* DK} \varepsilon_{\alpha\beta\gamma}(p, s) p'^\alpha p'^\beta p'^\gamma, \\
\langle D(p') K(q) | D_{s2}^*(p) \rangle &= G_{D_{s2}^* DK} \varepsilon_{\alpha\beta}(p, s) p'^\alpha p'^\beta, \\
\langle D(p') K(q) | D_{s1}^*(p) \rangle &= G_{D_{s1}^* DK} \varepsilon_\alpha(p, s) p'^\alpha, \\
\langle D(p') K(q) | D_{s0}^*(p) \rangle &= G_{D_{s0}^* DK}, \tag{13}
\end{aligned}$$

$$\begin{aligned}
\langle D^*(p')K(q) | D_{s3}^*(p) \rangle &= G_{D_{s3}^* D^* K} \varepsilon_{\alpha\beta\lambda\tau} \varepsilon^{*\alpha}(p', s') \varepsilon^{\beta\omega\theta}(p, s) p^\lambda p'^\tau p'_\omega p'_\theta, \\
\langle D^*(p')K(q) | D_{s2}^*(p) \rangle &= G_{D_{s2}^* D^* K} \varepsilon_{\alpha\beta\lambda\tau} \varepsilon^{*\alpha}(p', s') \varepsilon^{\beta\omega}(p, s) p^\lambda p'^\tau p'_\omega, \\
\langle D^*(p')K(q) | D_{s1}^*(p) \rangle &= G_{D_{s1}^* D^* K} \varepsilon_{\alpha\beta\lambda\tau} \varepsilon^{*\alpha}(p', s') \varepsilon^\beta(p, s) p^\lambda p'^\tau,
\end{aligned} \tag{14}$$

the $\varepsilon_{\mu\nu\rho}(p, s)$, $\varepsilon_{\mu\nu}(p, s)$ and $\varepsilon_\mu(p, s)$ are the mesons' polarization vectors.

Now we rewrite the correlation functions $\Pi_{\mu\nu\rho}(p, p')$ and $\Pi_{\sigma\mu\nu\rho}(p, p')$ at the phenomenological side into the following form,

$$\begin{aligned}
\Pi_{\mu\nu\rho}(p, p') &= \Pi_{DK,3}(p^2, p'^2) p'_\mu p'_\nu p'_\rho + \tilde{\Pi}_{DK,3}(p^2, p'^2) (p'_\mu g_{\nu\rho} + p'_\nu g_{\mu\rho} + p'_\rho g_{\mu\nu}) \\
&\quad + \Pi_{DK,3/2/1/0}(p^2, p'^2) p_\mu p_\nu p_\rho + \Pi_{DK,3/2}(p^2, p'^2) (p_\mu g_{\nu\rho} + p_\nu g_{\mu\rho} + p_\rho g_{\mu\nu}) \\
&\quad + \Pi_{DK,3/2/1}(p^2, p'^2) (p'_\mu p_\nu p_\rho + p'_\nu p_\mu p_\rho + p'_\rho p_\mu p_\nu) \\
&\quad + \Pi_{DK,3/2}(p^2, p'^2) (p'_\mu p'_\nu p_\rho + p'_\nu p'_\rho p_\mu + p'_\rho p'_\mu p_\nu),
\end{aligned} \tag{15}$$

$$\begin{aligned}
\Pi_{\sigma\mu\nu\rho}(p, p') &= \Pi_{D^*K,3}(p^2, p'^2) \frac{1}{3} (\varepsilon_{\sigma\rho\lambda\tau} p'_\mu p'_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p'_\mu p'_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p'_\nu p'_\rho p^\lambda p'^\tau) \\
&\quad + \tilde{\Pi}_{D^*K,3}(p^2, p'^2) (g_{\mu\nu} \varepsilon_{\sigma\rho\lambda\tau} p^\lambda p'^\tau + g_{\mu\rho} \varepsilon_{\sigma\nu\lambda\tau} p^\lambda p'^\tau + g_{\nu\rho} \varepsilon_{\sigma\mu\lambda\tau} p^\lambda p'^\tau) \\
&\quad + \Pi_{D^*K,3/2/1}(p^2, p'^2) (\varepsilon_{\sigma\rho\lambda\tau} p_\mu p_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p_\mu p_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p_\nu p_\rho p^\lambda p'^\tau) \\
&\quad + \Pi_{D^*K,3/2}(p^2, p'^2) (\varepsilon_{\sigma\rho\lambda\tau} p'_\mu p_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\rho\lambda\tau} p_\mu p'_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p'_\mu p_\rho p^\lambda p'^\tau \\
&\quad + \varepsilon_{\sigma\nu\lambda\tau} p_\mu p'_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p'_\nu p_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p_\nu p'_\rho p^\lambda p'^\tau),
\end{aligned} \tag{16}$$

so as to isolate the components associated with the special tensor structures which only receive contributions come from the spin-3 meson $D_{s3}^*(2860)$, where the contributions come from the higher resonances and continuum states are neglected, the subscripts 3, 2, 1 and 0 denote that there are contributions come from the $J^P = 3^-, 2^+, 1^-$ and 0^+ $c\bar{s}$ mesons, respectively. From Eqs.(15-16), we can see that the components $\Pi_{DK,3}(p^2, p'^2)$, $\tilde{\Pi}_{DK,3}(p^2, p'^2)$, $\Pi_{D^*K,3}(p^2, p'^2)$ and $\tilde{\Pi}_{D^*K,3}(p^2, p'^2)$ only receive contributions come from the spin-3 meson $D_{s3}^*(2860)$. The polarization vector $\varepsilon_{\mu\nu\rho}(p, s)$ satisfies the relation $g^{\mu\nu} \varepsilon_{\mu\nu\rho}(p, s) = g^{\mu\rho} \varepsilon_{\mu\nu\rho}(p, s) = g^{\nu\rho} \varepsilon_{\mu\nu\rho}(p, s) = 0$. If we multiply both sides of Eq.(5) by $g^{\mu\nu}$, we can obtain

$$g^{\mu\nu} \langle 0 | J_{\mu\nu\rho}(0) | D_{s3}^*(p) \rangle \neq f_{D_{s3}^*} g^{\mu\nu} \varepsilon_{\mu\nu\rho}(p, s) = 0, \tag{17}$$

the equation does not survive. We have to introduce the traceless current $\bar{J}_{\mu\nu\rho}$ by taking the following replacement,

$$J_{\mu\nu\rho} \rightarrow \bar{J}_{\mu\nu\rho} = J_{\mu\nu\rho} - \frac{1}{6} g_{\mu\nu} g^{\alpha\beta} J_{\alpha\beta\rho} - \frac{1}{6} g_{\mu\rho} g^{\alpha\beta} J_{\alpha\nu\beta} - \frac{1}{6} g_{\nu\rho} g^{\alpha\beta} J_{\mu\alpha\beta}, \tag{18}$$

then the traceless current $\bar{J}_{\mu\nu\rho}$ satisfies the relations $g^{\mu\nu} \bar{J}_{\mu\nu\rho} = g^{\mu\rho} \bar{J}_{\mu\nu\rho} = g^{\nu\rho} \bar{J}_{\mu\nu\rho} = 0$, and

$$\langle 0 | \bar{J}_{\mu\nu\rho}(0) | D_{s3}^*(p) \rangle = f_{D_{s3}^*} \varepsilon_{\mu\nu\rho}(p, s). \tag{19}$$

According to Eq.(5) and Eq.(19), we can choose either the current $J_{\mu\nu\rho}(x)$ or the current $\bar{J}_{\mu\nu\rho}(x)$ to interpolate the $D_{s3}^*(2860)$, as the components $\Pi_{DK,3}(p^2, p'^2)$, $\tilde{\Pi}_{DK,3}(p^2, p'^2)$, $\Pi_{D^*K,3}(p^2, p'^2)$ and $\tilde{\Pi}_{D^*K,3}(p^2, p'^2)$ at the phenomenological side are not changed. At the QCD side, if the current $\bar{J}_{\mu\nu\rho}(x)$ is chosen, the components $\Pi_{DK,3}(p^2, p'^2)$ and $\Pi_{D^*K,3}(p^2, p'^2)$ are not modified, but the components $\tilde{\Pi}_{DK,3}(p^2, p'^2)$ and $\tilde{\Pi}_{D^*K,3}(p^2, p'^2)$ are modified remarkably. In calculations, we observe that the components $\tilde{\Pi}_{DK,3}(p^2, p'^2)$ and $\tilde{\Pi}_{D^*K,3}(p^2, p'^2)$ cannot lead to reliable QCD sum rules and they are discarded. The pertinent tensor structures are $p'_\mu p'_\nu p'_\rho$ and $\varepsilon_{\sigma\rho\lambda\tau} p'_\mu p'_\nu p^\lambda p'^\tau + \varepsilon_{\sigma\nu\lambda\tau} p'_\mu p'_\rho p^\lambda p'^\tau + \varepsilon_{\sigma\mu\lambda\tau} p'_\nu p'_\rho p^\lambda p'^\tau$, we choose the two components $\Pi_{DK,3}(p^2, p'^2)$ and $\Pi_{D^*K,3}(p^2, p'^2)$ to study the hadronic coupling constants $G_{D_{s3}^* DK}$ and $G_{D_{s3}^* D^* K}$, respectively.

Now, we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu\rho}(p, p')$ and $\Pi_{\sigma\mu\nu\rho}(p, p')$ in perturbative QCD. We contract the quark fields in the correlation functions $\Pi_{\mu\nu\rho}(p, p')$ and $\Pi_{\sigma\mu\nu\rho}(p, p')$ with Wick theorem firstly,

$$\begin{aligned}\Pi_{\mu\nu\rho}(p, p') &= \int d^4x d^4y e^{ip' \cdot x} e^{i(p-p') \cdot (y-z)} \text{Tr} \{ i\gamma_5 U_{ij}(x-y) i\gamma_5 S_{jk}(y-z) \Gamma_{\mu\nu\rho} C_{ki}(z-x) \} |_{z=0}, \\ \Pi_{\sigma\mu\nu\rho}(p, p') &= \int d^4x d^4y e^{ip' \cdot x} e^{i(p-p') \cdot (y-z)} \text{Tr} \{ \gamma_\sigma U_{ij}(x-y) i\gamma_5 S_{jk}(y-z) \Gamma_{\mu\nu\rho} C_{ki}(z-x) \} |_{z=0},\end{aligned}\quad (20)$$

where

$$\Gamma_{\mu\nu\rho} = \gamma_\mu \frac{\overleftrightarrow{\partial}}{\partial z^\nu} \frac{\overleftrightarrow{\partial}}{\partial z^\rho} + \gamma_\nu \frac{\overleftrightarrow{\partial}}{\partial z^\mu} \frac{\overleftrightarrow{\partial}}{\partial z^\rho} + \gamma_\rho \frac{\overleftrightarrow{\partial}}{\partial z^\mu} \frac{\overleftrightarrow{\partial}}{\partial z^\nu}, \quad (21)$$

$$\begin{aligned}C_{ij}(x) &= \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\delta_{ij}}{\not{k} - m_c} - \frac{g_s G_{\alpha\beta}^n t_{ij}^n}{4} \frac{\sigma^{\alpha\beta} (\not{k} + m_c) + (\not{k} + m_c) \sigma^{\alpha\beta}}{(k^2 - m_c^2)^2} \right. \\ &\quad \left. + \frac{ig_s^2 G G \delta_{ij}}{12} \frac{m_c k^2 + m_c^2 \not{k}}{(k^2 - m_c^2)^4} + \dots \right\},\end{aligned}\quad (22)$$

$t^n = \frac{\lambda^n}{2}$, the λ^n is the Gell-Mann matrix, the i, j, k are color indexes [17]. We usually choose the full light quark propagators in the coordinate space. In the present case, the quark condensates and mixed condensates have no contributions, so we can take a simple replacement $c \rightarrow d/s$ to obtain the full d/s quark propagators. We compute all the integrals, then obtain the QCD spectral density through dispersion relation.

The leading-order contributions $\Pi_{\mu\nu\rho}^0(p, p')$ and $\Pi_{\sigma\mu\nu\rho}^0(p, p')$ can be written as

$$\begin{aligned}\Pi_{\mu\nu\rho}^0(p, p') &= \frac{3i}{(2\pi)^4} \int d^4k \frac{\text{Tr} \{ \gamma_5 [\not{k} + m_d] \gamma_5 [\not{k} + \not{p} - \not{p}' + m_s] \Gamma_{\mu\nu\rho} [\not{k} - \not{p}' + m_c] \}}{[k^2 - m_d^2] [(k+p-p')^2 - m_s^2] [(k-p')^2 - m_c^2]}, \\ &= \int ds du \frac{\rho_{\mu\nu\rho}(s, u)}{(s-p^2)(u-p'^2)},\end{aligned}\quad (23)$$

$$\begin{aligned}\Pi_{\sigma\mu\nu\rho}^0(p, p') &= \frac{3}{(2\pi)^4} \int d^4k \frac{\text{Tr} \{ \gamma_\sigma [\not{k} + m_d] \gamma_5 [\not{k} + \not{p} - \not{p}' + m_s] \Gamma_{\mu\nu\rho} [\not{k} - \not{p}' + m_c] \}}{[k^2 - m_d^2] [(k+p-p')^2 - m_s^2] [(k-p')^2 - m_c^2]}, \\ &= \int ds du \frac{\rho_{\sigma\mu\nu\rho}(s, u)}{(s-p^2)(u-p'^2)},\end{aligned}\quad (24)$$

where

$$\begin{aligned}\Gamma_{\mu\nu\rho} &= -\gamma_\mu (p-2k-2p')_\nu (p-2k-2p')_\rho - \gamma_\nu (p-2k-2p')_\mu (p-2k-2p')_\rho \\ &\quad - \gamma_\rho (p-2k-2p')_\mu (p-2k-2p')_\nu.\end{aligned}\quad (25)$$

The gluon field $G_\mu(z)$ in the covariant derivative has no contributions as $G_\mu(z) = \frac{1}{2} z^\lambda G_{\lambda\mu}(0) + \dots = 0$. We put all the quark lines on mass-shell by using the Cutkosky's rules, see Fig.1, and obtain the leading-order QCD spectral densities $\rho_{\mu\nu\rho}(s, u)$ and $\rho_{\sigma\mu\nu\rho}(s, u)$,

$$\begin{aligned}\rho_{\mu\nu\rho}(s, u) &= \frac{3}{(2\pi)^3} \int d^4k \delta[k^2 - m_d^2] \delta[(k+p-p')^2 - m_s^2] \delta[(k-p')^2 - m_c^2] \\ &\quad \text{Tr} \{ \gamma_5 [\not{k} + m_d] \gamma_5 [\not{k} + \not{p} - \not{p}' + m_s] \Gamma_{\mu\nu\rho} [\not{k} - \not{p}' + m_c] \},\end{aligned}\quad (26)$$

$$\begin{aligned}\rho_{\sigma\mu\nu\rho}(s, u) &= -\frac{3i}{(2\pi)^3} \int d^4k \delta[k^2 - m_d^2] \delta[(k+p-p')^2 - m_s^2] \delta[(k-p')^2 - m_c^2] \\ &\quad \text{Tr} \{ \gamma_\sigma [\not{k} + m_d] \gamma_5 [\not{k} + \not{p} - \not{p}' + m_s] \Gamma_{\mu\nu\rho} [\not{k} - \not{p}' + m_c] \}.\end{aligned}\quad (27)$$

It is straightforward to compute the integrals ², some useful identities are given explicitly in the appendix. The contributions of the gluon condensates shown in Fig.2 are calculated in the same way.

Once the analytical expressions of the QCD spectral densities are obtained, we can take quark-hadron duality below the continuum thresholds s_0 and u_0 respectively, and perform the double Borel transform with respect to the variables $P^2 = -p^2$ and $P'^2 = -p'^2$ to obtain the QCD sum rules,

$$\begin{aligned}\Pi_{DK,3}(M_1^2, M_2^2) &= -\frac{f_D M_D^2 f_K M_K^2 f_{D_{s3}^*} G_{D_{s3}^* DK}(q^2)}{(m_c + m_d)(m_d + m_s)(M_K^2 - q^2)} \exp\left(-\frac{M_{D_{s3}^*}^2}{M_1^2} - \frac{M_D^2}{M_2^2}\right) \\ &= \int ds du \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right) \frac{9}{4\pi^2 \sqrt{\lambda(s, u, q^2)}} \bar{\rho}_{DK},\end{aligned}\quad (28)$$

$$\begin{aligned}\Pi_{D^*K,3}(M_1^2, M_2^2) &= \frac{f_{D^*} M_{D^*} f_K M_K^2 f_{D_{s3}^*} G_{D_{s3}^* D^*K}(q^2)}{(m_d + m_s)(M_K^2 - q^2)} \exp\left(-\frac{M_{D_{s3}^*}^2}{M_1^2} - \frac{M_{D^*}^2}{M_2^2}\right) \\ &= \int ds du \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right) \frac{9}{4\pi^2 \sqrt{\lambda(s, u, q^2)}} \bar{\rho}_{D^*K},\end{aligned}\quad (29)$$

where

$$\begin{aligned}\int ds du &= \int_{m_c^2}^{s_0} ds \int_{m_c^2}^{u_0} du \quad |_{-1 \leq \cos \theta \leq 1}, \\ \cos \theta &= \frac{(u - q^2 - m_c^2)(s + u - q^2) - 2s(u - m_c^2)}{|u - q^2 - m_c^2| \sqrt{\lambda(u, s, q^2)}},\end{aligned}\quad (30)$$

² We choose the four-vectors as $p = (\sqrt{s}, 0)$, $p' = (p'_0, \vec{p}')$, $k = (k_0, \vec{k})$, and obtain the following solutions

$$\begin{aligned}k_0 &= \frac{u - q^2 + m_s^2 - m_c^2}{2\sqrt{s}}, \quad |\vec{k}| = \sqrt{\left(\frac{u - q^2 + m_s^2 - m_c^2}{2\sqrt{s}}\right)^2 - m_d^2}, \\ p'_0 &= \frac{s + u - q^2}{2\sqrt{s}}, \quad |\vec{p}'| = \frac{\sqrt{\lambda(s, u, q^2)}}{2\sqrt{s}},\end{aligned}$$

from the three Dirac δ -functions in Eq.(26) or Eq.(27). Then we obtain $\cos \theta$

$$\cos \theta = \frac{(u - q^2 + m_s^2 - m_c^2)(s + u - q^2) - 2s(u + m_d^2 - m_c^2)}{\sqrt{(u - q^2 + m_s^2 - m_c^2)^2 - 4sm_d^2} \sqrt{\lambda(s, u, q^2)}},$$

from the identity

$$(k - p')^2 - m_c^2 = m_d^2 + u - 2k_0 p'_0 + 2|\vec{k}||\vec{p}'| \cos \theta - m_c^2 = 0,$$

where we have used $\vec{k} \cdot \vec{p}' = |\vec{k}||\vec{p}'| \cos \theta$. If we take the approximation $m_d^2 \approx m_s^2 \approx 0$, then we obtain the constraint in Eq.(30).

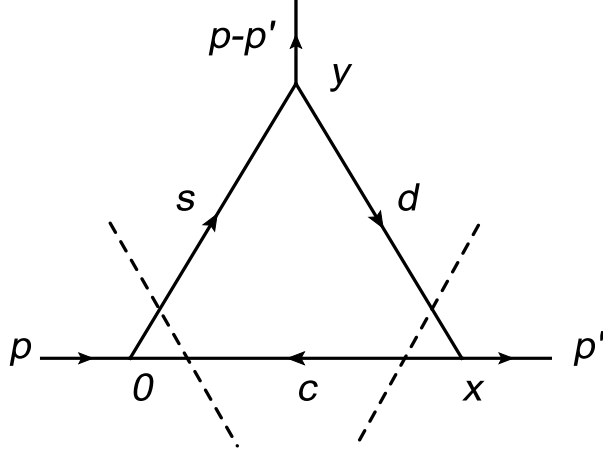


Figure 1: The leading-order contributions, the dashed lines denote the Cutkosky's cuts.

$$\begin{aligned}
\bar{\rho}_{DK} = & -2m_c^2 + 4m_d m_c + 2u + 2q^2 + b_1 (4m_c^2 - 12m_d m_c + 4m_s m_c + 2s - 6u - 6q^2) \\
& + b_2 (-2m_c^2 + 12m_d m_c - 8m_s m_c - 4s + 6u + 6q^2) \\
& + f_3 (-4m_d m_c + 4m_s m_c + 2s - 2u - 2q^2) \\
& + \frac{\pi^2}{9} \langle \frac{\alpha_s GG}{\pi} \rangle \left\{ 16 \frac{\partial b_1}{\partial m_B^2} + 16 \frac{\partial b_1}{\partial m_A^2} + 16 \frac{\partial b_1}{\partial m_c^2} - 17 \frac{\partial b_2}{\partial m_B^2} - 14 \frac{\partial b_2}{\partial m_A^2} - 17 \frac{\partial b_2}{\partial m_c^2} \right. \\
& + 6 \frac{\partial f_3}{\partial m_B^2} + 4 \frac{\partial f_3}{\partial m_A^2} + 6 \frac{\partial f_3}{\partial m_c^2} + (3u - m_c^2 - 2s + 9q^2) \frac{\partial^2 b_2}{\partial m_A^2 \partial m_B^2} \\
& + (s - u - 3q^2) \frac{\partial^2 f_3}{\partial m_A^2 \partial m_B^2} + (9u - 7m_c^2 - 2s + 3q^2) \frac{\partial^2 b_2}{\partial m_A^2 \partial m_c^2} \\
& + (2m_c^2 + s - 3u - q^2) \frac{\partial^2 f_3}{\partial m_A^2 \partial m_c^2} + (2s + 3u - 5m_c^2 + 3q^2) \frac{\partial^2 b_2}{\partial m_B^2 \partial m_c^2} \\
& \left. + (2m_c^2 - s - u - q^2) \frac{\partial^2 f_3}{\partial m_B^2 \partial m_c^2} - m_c^2 \frac{\partial^2 b_2}{\partial (m_c^2)^2} + m_c^2 (s - u - q^2) \frac{\partial^3 f_3}{\partial (m_c^2)^3} \right\}, \quad (31)
\end{aligned}$$

$$\begin{aligned}
\bar{\rho}_{D^*K} = & -4m_d + 4(m_s - m_c) a_1 - 4(m_c - 3m_d) b_1 + 4(2m_c - 3m_d) b_2 + 8(m_c - m_s) c_2 \\
& + 4(m_s - m_c) e_3 + 4(m_d - m_c) f_3 \\
& + \frac{2\pi^2}{9} m_c \langle \frac{\alpha_s GG}{\pi} \rangle \left\{ 2 \frac{\partial^2 b_2}{\partial m_A^2 \partial m_B^2} + 2 \frac{\partial^2 c_2}{\partial m_A^2 \partial m_B^2} - \frac{\partial^2 e_3}{\partial m_A^2 \partial m_B^2} - \frac{\partial^2 f_3}{\partial m_A^2 \partial m_B^2} \right. \\
& - 2 \frac{\partial^2 b_2}{\partial m_A^2 \partial m_c^2} - 2 \frac{\partial^2 c_2}{\partial m_A^2 \partial m_c^2} + \frac{\partial^2 e_3}{\partial m_A^2 \partial m_c^2} + \frac{\partial^2 f_3}{\partial m_A^2 \partial m_c^2} \\
& - 2 \frac{\partial^2 b_2}{\partial m_B^2 \partial m_c^2} - 2 \frac{\partial^2 c_2}{\partial m_B^2 \partial m_c^2} + \frac{\partial^2 e_3}{\partial m_B^2 \partial m_c^2} + \frac{\partial^2 f_3}{\partial m_B^2 \partial m_c^2} \\
& \left. + 2 \frac{\partial^2 b_2}{\partial (m_c^2)^2} + 2 \frac{\partial^2 c_2}{\partial (m_c^2)^2} - \frac{\partial^2 e_3}{\partial (m_c^2)^2} - \frac{\partial^2 f_3}{\partial (m_c^2)^2} - m_c^2 \frac{\partial^3 e_3}{\partial (m_c^2)^3} - m_c^2 \frac{\partial^3 f_3}{\partial (m_c^2)^3} \right\}, \quad (32)
\end{aligned}$$

the explicit expressions of the coefficients $a_1, b_1, b_2, c_2, e_3, f_3$ are given in the appendix.

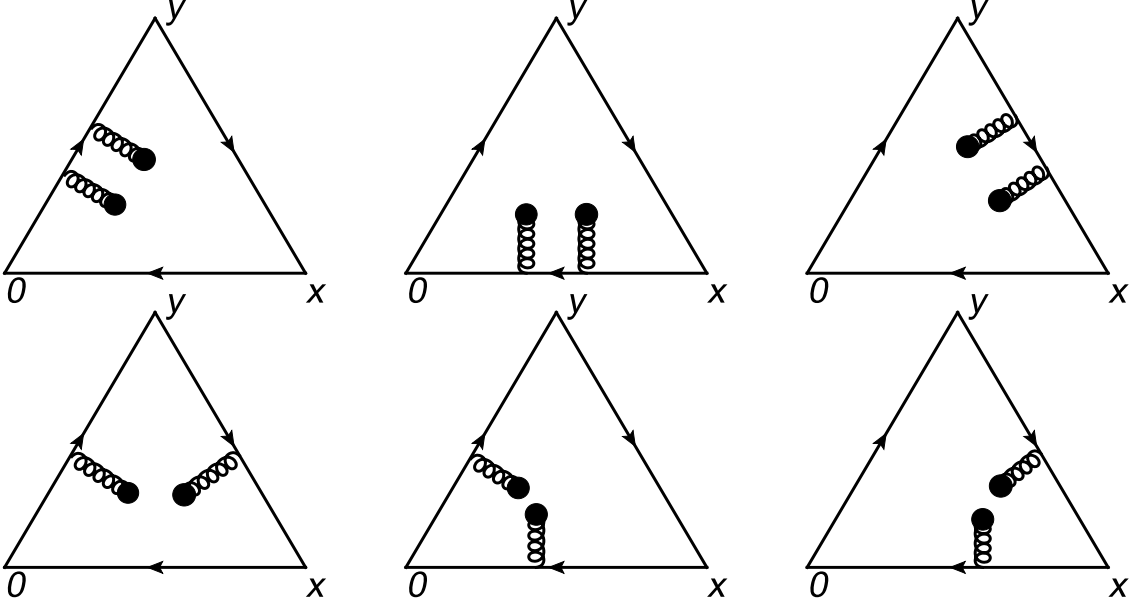


Figure 2: The gluon condensate contributions.

3 Numerical results and discussions

The value of the gluon condensate is taken to be the standard value $\langle \frac{\alpha_s GG}{\pi} \rangle = 0.012 \text{ GeV}^4$ [16, 17]. In the article, we take the \overline{MS} masses $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ and $m_s(\mu = 2 \text{ GeV}) = (0.095 \pm 0.005) \text{ GeV}$ from the Particle Data Group [26], and take into account the energy-scale dependence of the \overline{MS} masses from the renormalization group equation,

$$\begin{aligned}
 m_s(\mu) &= m_s(2 \text{ GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{4}{9}}, \\
 m_d(\mu) &= m_d(1 \text{ GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(1 \text{ GeV})} \right]^{\frac{4}{9}}, \\
 m_c(\mu) &= m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{25}}, \\
 \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \quad (33)
 \end{aligned}$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 213 \text{ MeV}$, 296 MeV and 339 MeV for the flavors $n_f = 5, 4$ and 3 , respectively [26]. Furthermore, we obtain the values $m_u = m_d = 6 \text{ MeV}$ from the Gell-Mann-Oakes-Renner relation at the energy scale $\mu = 1 \text{ GeV}$. In calculations, we take $n_f = 4$ and $\mu = \mu_{D_{s3}^*} = 2.1 \text{ GeV}$ [18, 27].

In Ref.[27], we study the masses and decay constants of the pseudoscalar, scalar, vector and axial-vector heavy mesons with the QCD sum rules in a systematic way. In this article, we take the values $M_D = 1.87 \text{ GeV}$, $M_{D^*} = 2.01 \text{ GeV}$, $f_D = 208 \text{ MeV}$, $f_{D^*} = 263 \text{ MeV}$, $M_2^2(D) = (1.2 - 1.8) \text{ GeV}^2$, $M_2^2(D^*) = (1.9 - 2.5) \text{ GeV}^2$, $u_0^D = (6.2 \pm 0.5) \text{ GeV}^2$, $u_0^{D^*} = (6.4 \pm 0.5) \text{ GeV}^2$ determined in the two-point QCD sum rules [27]. In Ref.[18], we assign the $D_{s3}^*(2860)$ to be a D-wave $c\bar{s}$ meson, and study the mass and decay constant (or current-meson coupling constant) of the $D_{s3}^*(2860)$ with the QCD sum rules by calculating the contributions of the vacuum condensates up to dimension-6 in the operator product expansion. In this article, we take the values $M_{D_{s3}^*} = 2.86 \text{ GeV}$, $f_{D_{s3}^*} =$

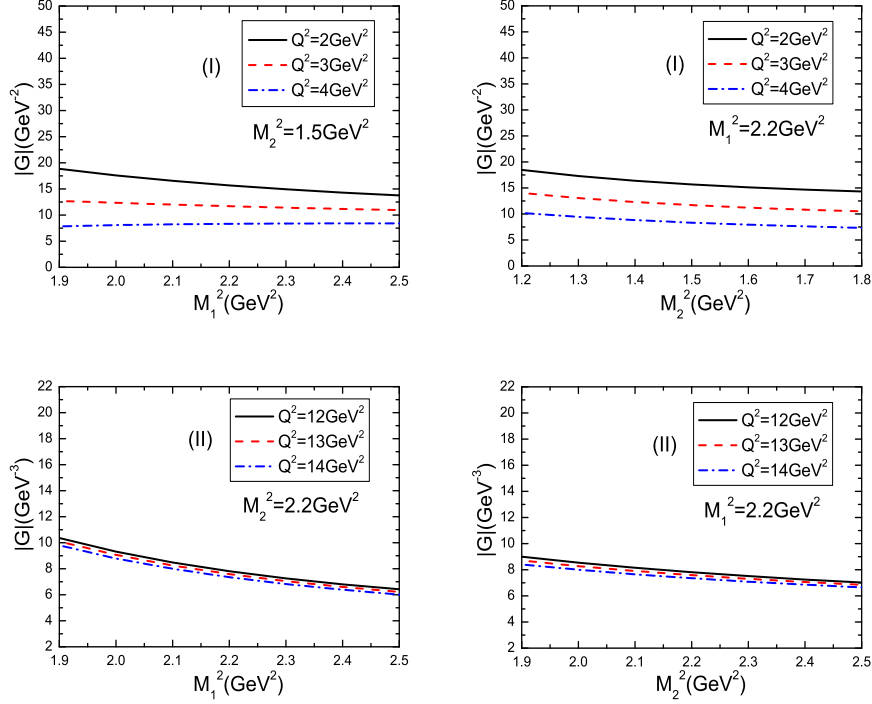


Figure 3: The hadronic coupling constants $G_{D_{s3}^* DK}(Q^2)$ (I) and $G_{D_{s3}^* D^* K}(Q^2)$ (II) with variations of the Borel parameters M_1^2 and M_2^2 , respectively.

6.02 GeV^4 , $M_1^2(D_{s3}^*) = (1.9 - 2.5) \text{ GeV}^2$, $s_0^{D_{s3}^*} = (11.6 \pm 0.7) \text{ GeV}^2$ determined in the two-point QCD sum rules [18]. Furthermore, we take the values $M_K = 0.495 \text{ GeV}$ and $f_K = 0.160 \text{ GeV}$ from the Particle Data Group [26]

In the following, we write down the definitions for the pole contributions of the $D_{s3}^*(2860)$, D and D^* in the QCD sum rules,

$$\begin{aligned} \text{pole}_{D_{s3}^*} &= \frac{\int_{m_c^2}^{s_0} ds \int_{m_c^2}^{\infty} du \rho_{QCD}(s, u) \big|_{-1 \leq \cos \theta \leq 1} \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right)}{\int_{m_c^2}^{\infty} ds \int_{m_c^2}^{\infty} du \rho_{QCD}(s, u) \big|_{-1 \leq \cos \theta \leq 1} \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right)}, \\ \text{pole}_{D/D^*} &= \frac{\int_{m_c^2}^{\infty} ds \int_{m_c^2}^{u_0} du \rho_{QCD}(s, u) \big|_{-1 \leq \cos \theta \leq 1} \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right)}{\int_{m_c^2}^{\infty} ds \int_{m_c^2}^{\infty} du \rho_{QCD}(s, u) \big|_{-1 \leq \cos \theta \leq 1} \exp\left(-\frac{s}{M_1^2} - \frac{u}{M_2^2}\right)}, \end{aligned} \quad (34)$$

where the $\rho_{QCD}(s, u)$ denotes the spectral densities at the QCD side. If we choose the Borel windows determined by the two-point QCD sum rules [18, 27], the pole contributions $\text{pole}_{D_{s3}^*/D/D^*} \gg 50\%$. For example, $\text{pole}_{D_{s3}^*} = (92-98)\%$ for $M_2^2 = 1.5 \text{ GeV}^2$ and $q^2 = -3 \text{ GeV}^2$; $\text{pole}_D = (86-98)\%$ for $M_1^2 = 2.2 \text{ GeV}^2$ and $q^2 = -3 \text{ GeV}^2$. The pole dominance is well satisfied. Moreover, in the Borel windows, the contributions come from the gluon condensate are of percent level, the operator product expansion is well convergent. The Borel windows determined by the two-point QCD sum rules still work in the three-point QCD sum rules, and we expect to make reasonable predictions.

In Fig.3, we plot the hadronic coupling constants $G_{D_{s3}^* DK}(Q^2)$ and $G_{D_{s3}^* D^* K}(Q^2)$ with variations of the Borel parameters M_1^2 and M_2^2 , where $Q^2 = -q^2$. From the figure, we can see that the values are not very stable with variations of the Borel parameters M_1^2 and M_2^2 . From the QCD sum

rules in Eqs.(28-32) or the explicit expressions of the $\bar{\rho}_{DK}$ and $\bar{\rho}_{D^*K}$, we can see that there are no contributions come from the quark condensates and mixed condensates, and no terms of the orders $\mathcal{O}\left(\frac{1}{M_1^2}\right)$, $\mathcal{O}\left(\frac{1}{M_2^2}\right)$, $\mathcal{O}\left(\frac{1}{M_1^4}\right)$, $\mathcal{O}\left(\frac{1}{M_2^4}\right)$, \dots , which are needed to stabilize the QCD sum rules so as to warrant a platform. The uncertainties originate from the Borel parameters are rather large, we take them into account. In calculations, we observe that the values of the $|G_{D_{s3}^*DK}(Q^2)|$ at the region $Q^2 > 1 \text{ GeV}^2$ decrease monotonously with increase of the Q^2 , while the values $G_{D_{s3}^*D^*K}(Q^2)$ change sign at the region $Q^2 = (1 - 2) \text{ GeV}^2$, we have to postpone the Q^2 to large values.

Now we fit the central values of the hadronic coupling constants $G_{D_{s3}^*DK}(Q^2)$ at $Q^2 = (2 - 4) \text{ GeV}^2$ and $G_{D_{s3}^*D^*K}(Q^2)$ at $Q^2 = (12 - 14) \text{ GeV}^2$ into the functions of the form $A + BQ^2$,

$$|G_{D_{s3}^*DK}(Q^2)| = 22.88 \text{ GeV}^{-2} - 3.69 Q^2 \text{ GeV}^{-4}, \quad (35)$$

$$|G_{D_{s3}^*D^*K}(Q^2)| = 10.61 \text{ GeV}^{-3} - 0.23 Q^2 \text{ GeV}^{-5}, \quad (36)$$

then we extend the values to the physical region $Q^2 = -M_K^2$, and obtain

$$|G_{D_{s3}^*DK}(Q^2 = -M_K^2)| = 23.8 \text{ GeV}^{-2}, \quad (37)$$

$$|G_{D_{s3}^*D^*K}(Q^2 = -M_K^2)| = 10.7 \text{ GeV}^{-3}, \quad (38)$$

the uncertainties of the $G_{D_{s3}^*DK}(Q^2 = -M_K^2)$ and $G_{D_{s3}^*D^*K}(Q^2 = -M_K^2)$ are about 18% and 28%, respectively.

We can take the physical values of the hadronic coupling constants $G_{D_{s3}^*DK}$ and $G_{D_{s3}^*D^*K}$ as input parameters and study the two-body strong decays, which take place through relative F-wave,

$$\begin{aligned} \Gamma(D_{s3}^*(2860) \rightarrow D^+ K^0 + D^0 K^+) &= \frac{1}{140\pi M_{D_{s3}^*}^2} G_{D_{s3}^*DK}^2 p^7 \times 2, \\ &= 28.3 \pm 10.2 \text{ MeV}, \end{aligned} \quad (39)$$

$$\begin{aligned} \Gamma(D_{s3}^*(2860) \rightarrow D^{*+} K^0 + D^{*0} K^+) &= \frac{1}{105\pi} G_{D_{s3}^*D^*K}^2 p'^7 \times 2, \\ &= 16.2 \pm 9.1 \text{ MeV}, \end{aligned} \quad (40)$$

where

$$\begin{aligned} p &= \frac{\sqrt{\lambda(M_{D_{s3}^*}^2, M_D^2, M_K^2)}}{2M_{D_{s3}^*}} = 709 \text{ MeV}, \\ p' &= \frac{\sqrt{\lambda(M_{D_{s3}^*}^2, M_{D^*}^2, M_K^2)}}{2M_{D_{s3}^*}} = 585 \text{ MeV}. \end{aligned} \quad (41)$$

If we saturate the decay width of the $D_{s3}^*(2860)$ with the strong decays to the final states $D^+ K^0$, $D^0 K^+$, $D^{*+} K^0$, $D^{*0} K^+$, the total decay width is $44.5 \pm 10.2 \pm 9.1 \text{ MeV}$, which is compatible with the width $\Gamma_{D_{s3}^*} = (53 \pm 7 \pm 4 \pm 6) \text{ MeV}$ from the LHCb collaboration [13, 14]. The predicted ratio R

$$R = \frac{\Gamma(D_{s3}^*(2860) \rightarrow D^* K)}{\Gamma(D_{s3}^*(2860) \rightarrow DK)} = 0.57 \pm 0.38, \quad (42)$$

which has minor overlap with the experimental value,

$$R = \frac{\text{Br}(D_{sJ}^*(2860) \rightarrow D^* K)}{\text{Br}(D_{sJ}^*(2860) \rightarrow DK)} = 1.10 \pm 0.15 \pm 0.19, \quad (43)$$

from the BaBar collaboration [2] due to the uncertainties, while the central value is much smaller than the experimental value. If we assign the $D_{sJ}^*(2860)$ to be the $D_{s3}^*(2860)$, the theoretical values R from the leading order heavy meson effective theory [3], the constituent quark model with quark-meson effective Lagrangians [5], the 3P_0 model [6, 9, 19, 20, 21] and the pseudoscalar emission decay model [22] are much smaller than the experimental value, see Table 1. If we take into account the chiral symmetry breaking corrections, the experimental value can be reproduced with suitable parameters in heavy meson effective theory [24]. In the present work, we cannot reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$ based on the QCD sum rules, and fail to obtain additional support for assigning the $D_{sJ}^*(2860)$ to be the $D_{s3}^*(2860)$.

We have two choices to reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$, one choice is taking into account the chiral symmetry breaking corrections by fitting the relevant parameters in the heavy meson effective Lagrangians [24]; the other choice is introducing some $D_{s2}(2860)$ and $D'_{s2}(2860)$ components in the $D_{sJ}^*(2860)$ beyond the $D_{s3}^*(2860)$ and the $D_{s1}^*(2860)$. The $J^P = 2^-$ mesons $D_{s2}(2860)$ and $D'_{s2}(2860)$ decay only to the final states D^*K . If the $D_{sJ}^*(2860)$ consists of at least four resonances $D_{s1}^*(2860)$, $D_{s2}(2860)$, $D'_{s2}(2860)$, $D_{s3}^*(2860)$, the large ratio $R = 1.10 \pm 0.15 \pm 0.19$ is easy to account for, as the components $D_{s2}(2860)$ and $D'_{s2}(2860)$ can enhance the branching fraction $\text{Br}(D_{sJ}^*(2860) \rightarrow D^*K)$ efficaciously.

4 Conclusion

In this article, we assign the $D_{s3}^*(2860)$ to be a D-wave $c\bar{s}$ meson, study the vertices $D_{s3}^*(2860)DK$ and $D_{s3}^*(2860)D^*K$ in details to select the pertinent tensor structures, then calculate the hadronic coupling constants $G_{D_{s3}^*(2860)DK}$ and $G_{D_{s3}^*(2860)D^*K}$ with the three-point QCD sum rules. Finally we obtain the partial decay widths $\Gamma(D_{s3}^*(2860) \rightarrow D^*K)$ and $\Gamma(D_{s3}^*(2860) \rightarrow DK)$, and the ratio $R = \Gamma(D_{s3}^*(2860) \rightarrow D^*K) / \Gamma(D_{s3}^*(2860) \rightarrow DK) = 0.57 \pm 0.38$. The predicted ratio $R = 0.57 \pm 0.38$ cannot reproduce the experimental value $R = 1.10 \pm 0.15 \pm 0.19$, although the theoretical and experimental values overlap slightly with each other due to the uncertainties. Some components $D_{s2}(2860)$ and $D'_{s2}(2860)$ are needed to reproduce the experimental value, if one would like not to resort to the chiral symmetry breaking corrections to dispel the discrepancy.

Appendix

The explicit expressions of the coefficients $a_1, b_1, a_2, b_2, c_2, d_2, a_3, b_3, c_3, d_3, e_3, f_3$ and

$$\begin{aligned} \frac{\partial}{\partial m_i^2} f &\doteq \frac{\partial}{\partial m_i^2} f(m_A, m_B, m_c) \big|_{m_A=0; m_B=0}, \\ \frac{\partial^2}{\partial m_i^2 \partial m_j^2} f &\doteq \frac{\partial^2}{\partial m_i^2 \partial m_j^2} f(m_A, m_B, m_c) \big|_{m_A=0; m_B=0}, \\ \frac{\partial^3}{\partial m_i^2 \partial m_j^2 \partial m_k^2} f &\doteq \frac{\partial^2}{\partial m_i^2 \partial m_j^2 \partial m_k^2} f(m_A, m_B, m_c) \big|_{m_A=0; m_B=0}, \end{aligned} \quad (44)$$

with $f(m_A, m_B, m_c) = a_1(m_A, m_B, m_c), b_1(m_A, m_B, m_c), a_2(m_A, m_B, m_c), b_2(m_A, m_B, m_c), \dots, m_i^2, m_j^2, m_k^2 = m_A^2, m_B^2, m_c^2$.

$$\begin{aligned}
\int d^4k \delta^3 &= \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}}, \\
\int d^4k \delta^3 k_\mu &= \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}} [a_1(m_A, m_B, m_c) p_\mu + b_1(m_A, m_B, m_c) p'_\mu], \\
\int d^4k \delta^3 k_\mu k_\nu &= \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}} [a_2(m_A, m_B, m_c) p_\mu p_\nu + b_2(m_A, m_B, m_c) p'_\mu p'_\nu \\
&\quad + c_2(m_A, m_B, m_c) (p_\mu p'_\nu + p'_\mu p_\nu) + d_2(m_A, m_B, m_c) g_{\mu\nu}], \\
\int d^4k \delta^3 k_\mu k_\nu k_\rho &= \frac{\pi}{2\sqrt{\lambda(s, u, q^2)}} [a_3(m_A, m_B, m_c) p_\mu p_\nu p_\rho \\
&\quad + b_3(m_A, m_B, m_c) (p_\mu g_{\nu\rho} + p_\nu g_{\mu\rho} + p_\rho g_{\mu\nu}) \\
&\quad + c_3(m_A, m_B, m_c) (p'_\mu g_{\nu\rho} + p'_\nu g_{\mu\rho} + p'_\rho g_{\mu\nu}) \\
&\quad + d_3(m_A, m_B, m_c) (p'_\mu p_\nu p_\rho + p'_\nu p_\mu p_\rho + p'_\rho p_\mu p_\nu) \\
&\quad + e_3(m_A, m_B, m_c) (p'_\mu p'_\nu p_\rho + p'_\nu p'_\rho p_\mu + p'_\rho p'_\mu p_\nu) \\
&\quad + f_3(m_A, m_B, m_c) p'_\mu p'_\nu p'_\rho], \tag{45}
\end{aligned}$$

$$\delta^3 = \delta[k^2 - m_A^2] \delta[(k + p - p')^2 - m_B^2] \delta[(k - p')^2 - m_c^2], \tag{46}$$

$$\begin{aligned}
a_1(m_A, m_B, m_c) &= \frac{1}{\lambda(s, u, q^2)} [m_c^2(u - s + q^2) + u(s - u + q^2) - 2um_B^2 \\
&\quad + m_A^2(u + s - q^2)], \\
b_1(m_A, m_B, m_c) &= \frac{1}{\lambda(s, u, q^2)} [m_c^2(s - u + q^2) + u(u - s - 2q^2) + q^2(q^2 - s) \\
&\quad - 2sm_A^2 + m_B^2(u + s - q^2)], \tag{47}
\end{aligned}$$

$$\begin{aligned}
a_2(m_A, m_B, m_c) &= \frac{1}{\lambda(s, u, q^2)} [(u - m_c^2)^2 - 2m_A^2(u + m_c^2)] \\
&\quad + \frac{6u}{\lambda^2(s, u, q^2)} \{ q^2 [m_c^4 - (u + s - q^2)m_c^2 + su] + m_A^2 m_B^2 (q^2 - u - s) \\
&\quad - m_A^2 [s(u - s + q^2) + m_c^2(s - u + q^2)] \\
&\quad - m_B^2 [u(s - u + q^2) + m_c^2(u - s + q^2)] \} , \\
b_2(m_A, m_B, m_c) &= \frac{1}{\lambda(s, u, q^2)} [(u - q^2 - m_c^2)^2 + 2m_B^2(u - q^2 - m_c^2) - 4sm_A^2] \\
&\quad + \frac{6s}{\lambda^2(s, u, q^2)} \{ q^2 [m_c^4 - (u + s - q^2)m_c^2 + su] + m_A^2 m_B^2 (q^2 - u - s) \\
&\quad + m_A^2 [s(s - u - q^2) + m_c^2(u - s - q^2)] \\
&\quad + m_B^2 [u(u - s - q^2) + m_c^2(s - u - q^2)] \} , \\
c_2(m_A, m_B, m_c) &= \frac{1}{\lambda(s, u, q^2)} [(u - m_c^2)(m_c^2 + q^2 - u) + m_B^2(m_c^2 - u) \\
&\quad + m_A^2(m_c^2 - q^2 - m_B^2 + 2s + u)] \\
&\quad - \frac{3(u + s - q^2)}{\lambda^2(s, u, q^2)} \{ q^2 [m_c^4 - (u + s - q^2)m_c^2 + su] + m_A^2 m_B^2 (q^2 - u - s) \\
&\quad - m_B^2 [m_c^2(u - s + q^2) + u(s - u + q^2)] \\
&\quad - m_A^2 [m_c^2(s - u + q^2) + s(u - s + q^2)] \} , \\
d_2(m_A, m_B, m_c) &= \frac{1}{2\lambda(s, u, q^2)} \{ q^2 [m_c^4 - (u + s - q^2)m_c^2 + su] + m_A^2 m_B^2 (q^2 - u - s) \\
&\quad + m_A^2 [s(s - u - q^2) + m_c^2(u - s - q^2)] \\
&\quad + m_B^2 [u(u - s - q^2) + m_c^2(s - u - q^2)] \} , \tag{48}
\end{aligned}$$

$$\begin{aligned}
a_3(0, 0, m_c) &= \frac{1}{\lambda^3(s, u, q^2)} \{ (m_c^2 - u)^3(u - s)^3 + 3(m_c^2 - u)^2(u - s)(u^2 + 3um_c^2 - 3us - sm_c^2)q^2 \\
&\quad - 3(m_c^2 - u) [m_c^4(s - 3u) + 6um_c^2(s - u) + u^2(3s - u)] q^4 \\
&\quad + (m_c^6 + 9um_c^4 + 9u^2m_c^2 + u^3) q^6 \} , \\
b_3(0, 0, m_c) &= \frac{1}{2\lambda^2(s, u, q^2)} \{ (s - m_c^2)(m_c^2 - u)^2(s - u)q^2 + (m_c^2 - u)(m_c^4 - 2sm_c^2 + 2um_c^2 - su) \\
&\quad q^4 + m_c^2(m_c^2 + u)q^6 \} , \\
c_3(0, 0, m_c) &= \frac{1}{2\lambda^2(s, u, q^2)} \{ (m_c^2 - s)(m_c^2 - u)^2(s - u)q^2 + (m_c^2 - u) [m_c^4 - (s + 3u)m_c^2 \\
&\quad + s(s + 2u)] q^4 + (2m_c^4 - 2sm_c^2 - 3um_c^2 + su) q^6 + m_c^2 q^8 \} , \tag{49}
\end{aligned}$$

$$\begin{aligned}
d_3(0, 0, m_c) &= \frac{1}{\lambda^3(s, u, q^2)} \{ (m_c^2 - u)^3(s - u)^3 + (m_c^2 - u)^2(s - u) [4u^2 + m_c^2(s + 5u) - 7us - 3s^2] \\
&\quad q^2 + (m_c^2 - u) [m_c^4(3u - 5s) + m_c^2(9s^2 - 2us - 15u^2) + u(3s^2 + 13us - 6u^2)] q^4 \\
&\quad + [3m_c^6 + m_c^4(6u - 9s) - 3um_c^2(2s + 5u) + u^2(5s - 4u)] q^6 \\
&\quad + (3m_c^4 + 6um_c^2 + u^2) q^8 \} , \tag{50}
\end{aligned}$$

$$\begin{aligned}
e_3(0, 0, m_c) &= \frac{1}{\lambda^3(s, u, q^2)} \{ (m_c^2 - u)^3 (u - s)^3 + (m_c^2 - u)^2 (u - s) [5u^2 + m_c^2(u + 5s) - 5us - 6s^2] \\
&\quad q^2 + [m_c^6(3s - 5u) + m_c^4(9u^2 + 15us - 6s^2) - 3m_c^2(s^3 + us^2 + 10u^2s - 2u^3) \\
&\quad + u(3s^3 + 9us^2 + 12u^2s - 10u^3)] q^4 \\
&\quad + [3m_c^6 - 3m_c^4(5u + 2s) + 3m_c^2(3s^2 + 8us + 2u^2) + u(10u^2 - 6us - 5s^2)] q^6 \\
&\quad + [6m_c^4 - 9m_c^2(u + s) + u(s - 5u)] q^8 + (u + 3m_c^2)q^{10} \} , \tag{51}
\end{aligned}$$

$$\begin{aligned}
f_3(0, 0, m_c) &= \frac{1}{\lambda^3(s, u, q^2)} \{ (m_c^2 - u)^3 (s - u)^3 + 3(m_c^2 - u)^2 (u - s) [3s^2 + us - 2u^2 + m_c^2(u - 3s)] \\
&\quad q^2 + 3 [m_c^6(3s - u) + m_c^4(6u^2 - 13us - 3s^2) + m_c^2(3s^3 + 9us^2 + 12u^2s - 10u^3) \\
&\quad + u(5u^3 - 2u^2s - 6us^2 - 3s^3)] q^4 \\
&\quad + [m_c^6 + 3m_c^4(5s - 4u) - 3m_c^2(5s^2 + 6us - 10u^2) + 6s^2u - 6su^2 - 20u^3 - s^3] q^6 \\
&\quad + 3 [m_c^4 + m_c^2(s - 5u) + s^2 + 3su + 5u^2] q^8 + 3(m_c^2 - s - 2u)q^{10} + q^{12} \} , \tag{52}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f_3}{\partial m_A^2} &= \frac{6s}{\lambda^3(s, u, q^2)} \{ (m_c^2 - u)(s - u)^2(u + s - 2m_c^2) - [s^3 + 4us^2 + u^2s - 4u^3 + m_c^4(6s - 4u) \\
&\quad + m_c^2(9u^2 - 5s^2 - 8us)] q^2 + [9um_c^2 - 2m_c^4 - 3sm_c^2 + s^2 - 6u^2 - us] q^4 \\
&\quad + (s + 4u - 3m_c^2) q^6 - q^8 \} , \\
\frac{\partial f_3}{\partial m_B^2} &= \frac{3}{\lambda^3(s, u, q^2)} \{ (m_c^2 - u)^2(s - u)^2(3s + u) + [m_c^4(3s^2 + 4us - 3u^2) \\
&\quad + m_c^2(2u^2s - 6s^3 - 12us^2 + 8u^3) + u(6s^3 + 9us^2 - 6u^2s - 5u^3)] q^2 \\
&\quad + [s^3 - us^2 + 12u^2s + 10u^3 + m_c^4(3u - 5s) + 2m_c^2(5s^2 + us - 6u^2)] q^4 \\
&\quad - [m_c^4 + 2m_c^2(s - 4u) + 3s^2 + 10u^2 + 10us] q^6 + (3s + 5u - 2m_c^2)q^8 - q^{10} \} , \tag{53}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 e_3}{\partial m_A^2 \partial m_B^2} &= \frac{2}{\lambda^3(s, u, q^2)} \{ (u - s) [s^3 + 12us^2 + 15u^2s + 2u^3 - 3m_c^2(3s^2 + 6us + u^2)] \\
&\quad + [s^3 - 4us^2 - 31u^2s - 8u^3 + m_c^2(9u^2 + 12us - 15s^2)] q^2 \\
&\quad + [3s^2 + 23us + 12u^2 + 3m_c^2(s - 3u)] q^4 + (3m_c^2 - 5s - 8u)q^6 + 2q^8 \} , \\
\frac{\partial^2 e_3}{\partial m_A^2 \partial m_c^2} &= \frac{2}{\lambda^3(s, u, q^2)} \{ -(s - u)^2 [2(s^2 + 4us + u^2) - 3m_c^2(u + 3s)] \\
&\quad + [9s^2m_c^2 - 4s^3 - 20us^2 + 4u^2s + 12usm_c^2 + 8u^3 - 9u^2m_c^2] q^2 \\
&\quad + [12s^2 - 15sm_c^2 + 4us - 12u^2 + 9um_c^2] q^4 + (8u - 4s - 3m_c^2)q^6 - 2q^8 \} , \\
\frac{\partial^2 e_3}{\partial m_B^2 \partial m_c^2} &= \frac{6}{\lambda^3(s, u, q^2)} \{ 2(u - m_c^2)(s - u)^2(u + s) \\
&\quad + [s^3 + 3us^2 + 5u^2s - 5u^3 + 2m_c^2(s^2 - 4us + u^2)] q^2 \\
&\quad + [2m_c^2(u + s) - 3(s^2 + 2us - u^2)] q^4 + (u + 3s - 2m_c^2)q^6 - q^8 \} , \tag{54}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 f_3}{\partial m_A^2 \partial m_B^2} &= \frac{6s}{\lambda^3(s, u, q^2)} \left\{ (s-u) [s^2 - 6sm_c^2 + 6us + 3u^2 - 4um_c^2] \right. \\
&\quad \left. + [s^2 + 8us + 9u^2 + 2m_c^2(s-4u)] q^2 + (4m_c^2 - 5s - 9u) q^4 + 3q^6 \right\}, \\
\frac{\partial^2 f_3}{\partial m_A^2 \partial m_c^2} &= \frac{6s}{\lambda^3(s, u, q^2)} \left\{ (s-u)^2 (s+3u-4m_c^2) + (5s^2 - 12sm_c^2 + 8us - 9u^2 + 8um_c^2) q^2 \right. \\
&\quad \left. + (9u - 3s - 4m_c^2) q^4 - 3q^6 \right\}, \\
\frac{\partial^2 f_3}{\partial m_B^2 \partial m_c^2} &= \frac{6}{\lambda^3(s, u, q^2)} \left\{ (m_c^2 - u)(s-u)^2 (3s+u) \right. \\
&\quad \left. + [u^2 s - 3s^3 - 6us^2 + 4u^3 + m_c^2(3s^2 + 4us - 3u^2)] q^2 \right. \\
&\quad \left. + (5s^2 - 5sm_c^2 + us - 6u^2 + 3um_c^2) q^4 + (4u - s - m_c^2) q^6 - q^8 \right\}, \tag{55}
\end{aligned}$$

here we have neglected the terms m_A^4 and m_B^4 in the a_2 , b_2 , c_2 and d_2 as they are irreverent in present calculations.

Acknowledgements

This work is supported by National Natural Science Foundation, Grant Numbers 11375063, and Natural Science Foundation of Hebei province, Grant Number A2014502017.

References

- [1] B. Aubert et al, Phys. Rev. Lett. **97** (2006) 222001.
- [2] B. Aubert et al, Phys. Rev. **D80** (2009) 092003.
- [3] P. Colangelo, F. De Fazio and S. Nicotri, Phys. Lett. **B642** (2006) 48.
- [4] X. H. Zhong and Q. Zhao, Phys. Rev. **D78** (2008) 014029.
- [5] X. H. Zhong and Q. Zhao, Phys. Rev. **D81** (2010) 014031.
- [6] B. Zhang, X. Liu, W. Z. Deng and S. L. Zhu, Eur. Phys. J. **C50** (2007) 617.
- [7] D. M. Li, B. Ma and Y. H. Liu, Eur. Phys. J. **C51** (2007) 359.
- [8] B. Chen, D. X. Wang and A. Zhang, Phys. Rev. **D80** (2009) 071502.
- [9] D. M. Li and B. Ma, Phys. Rev. **D81** (2010) 014021.
- [10] A. M. Badalian and B. L. G. Bakker, Phys. Rev. **D84** (2011) 034006.
- [11] J. Vijande, A. Valcarce and F. Fernandez, Phys. Rev. **D79** (2009) 037501.
- [12] F. K. Guo and U. G. Meissner, Phys. Rev. **D84** (2011) 014013.
- [13] R. Aaij et al, Phys. Rev. Lett. **113** (2014) 162001.
- [14] R. Aaij et al, Phys. Rev. **D90** (2014) 072003.
- [15] R. Aaij et al, JHEP **1210** (2012) 151.
- [16] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385; Nucl. Phys. **B147** (1979) 448.

- [17] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1.
- [18] Z. G. Wang, arXiv:1603.06026.
- [19] Q. T. Song, D. Y. Chen, X. Liu and T. Matsuki, Eur. Phys. J. **C75** (2015) 30.
- [20] J. Segovia, D. R. Entem and F. Fernandez, Phys. Rev. **D91** (2015) 094020.
- [21] B. Chen, X. Liu and A. Zhang, Phys. Rev. **D92** (2015) 034005.
- [22] S. Godfrey and I. T. Jardine, Phys. Rev. **D89** (2014) 074023.
- [23] A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **10** (2000) 1; M. Neubert, Phys. Rept. **245** (1994) 259.
- [24] Z. G. Wang, Eur. Phys. J. **C75** (2015) 25.
- [25] J. J. Zhu and M. L. Yan, hep-ph/9903349; and references therein.
- [26] K. A. Olive et al, Chin. Phys. **C38** (2014) 090001.
- [27] Z. G. Wang, Eur. Phys. J. **C75** (2015) 427.